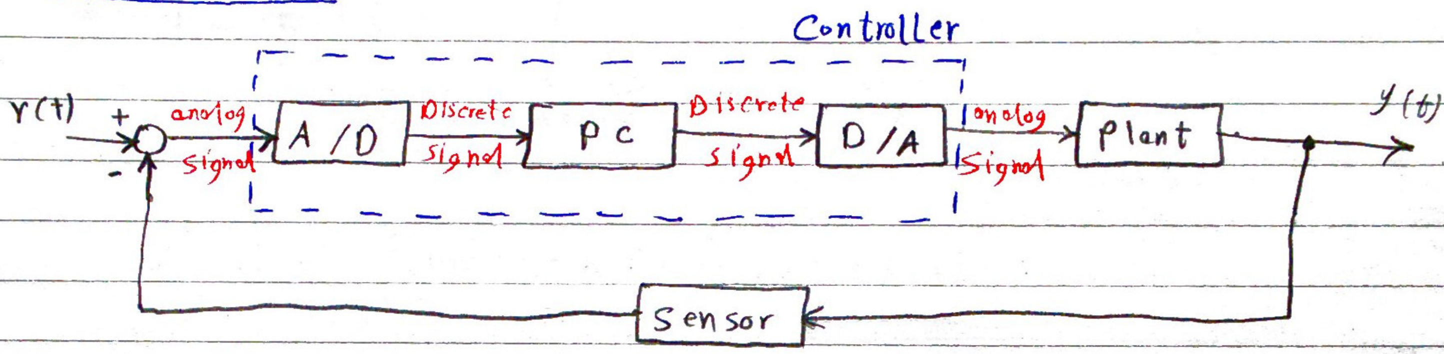
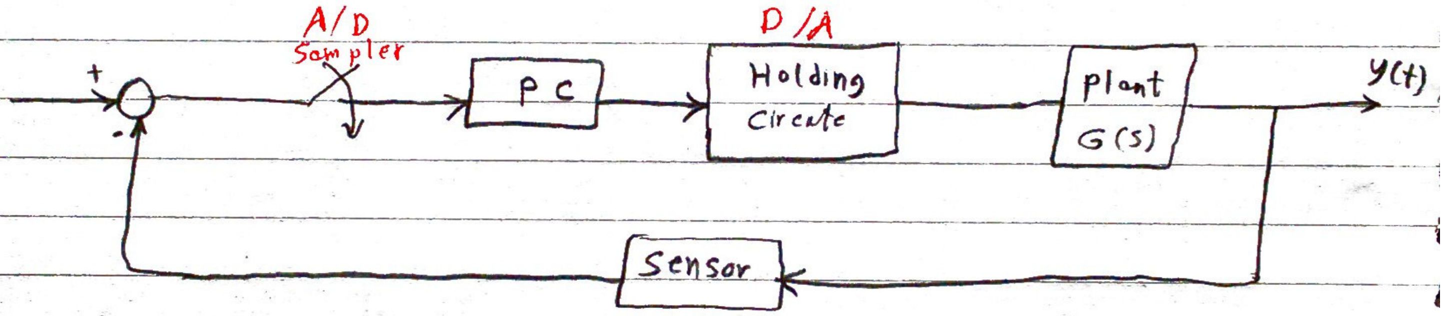
Lec 1

Revision on Z.T.

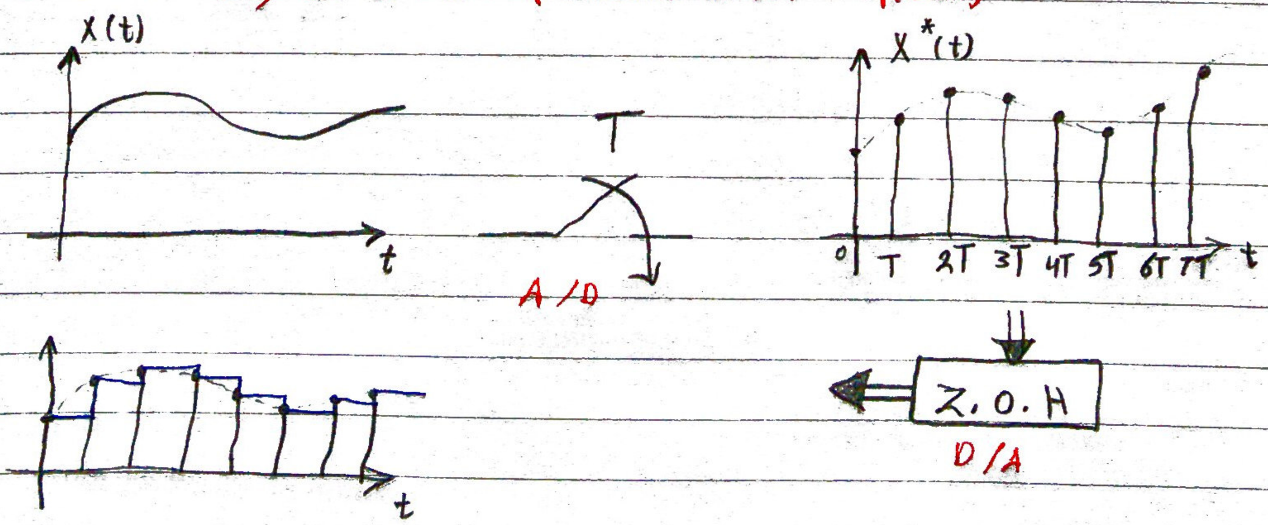
Digital system:





any Digital System Contains:

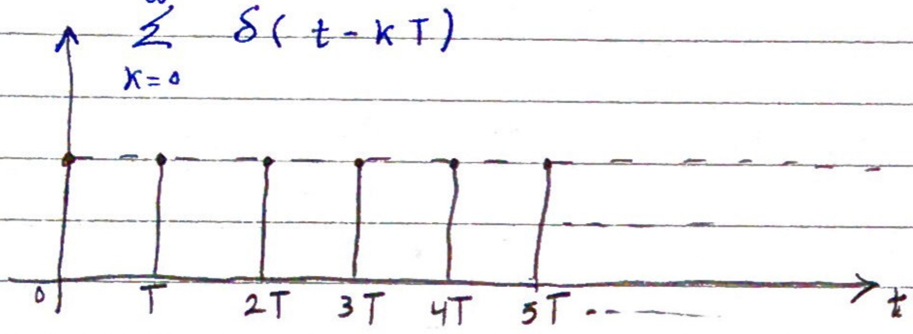
1. A/D
2. D/A Converter ≡ holding Circuite
   - Z.O.H (Zero order hold)
   - F.O.H (First order hold)
   - S.O.H (Second order hold)



Can be smothed using L.P.F

The sampler can be viewed

$$\sum_{K=0}^{\infty} \delta(t-KT)$$



$$x^*(t) = \sum_{K=0}^{\infty} x(t)\delta(t-KT) = x(0)+x(T)+x(2T)+\cdots$$

$K \rightarrow$ Sample number

$T \rightarrow$ sample period

$x^*(t) \rightarrow$ output of the sampler

$$x^*(t) = \sum_{K=0}^{\infty} x(t)\,\delta(t-KT) = \sum_{K=0}^{\infty} x(KT)\,\delta(t-KT)$$

$\rightarrow$ Laplace Transform

$$x^*(s) = \sum_{K=0}^{\infty} x(KT)\, e^{-KTs}$$

$\rightarrow$ Z. Trans Form $\qquad z = e^{Ts}$

$$x(z) = \sum_{K=0}^{\infty} x(KT)\, z^{-K}$$

$$x(t) \xrightarrow{Z.T.} x(z) = \sum_{K=0}^{\infty} x(KT)\, z^{-K}$$

Ex: $x(t) = u(t) = 1$

$$X(z) = \sum_{k=0}^{\infty} z^{-k} = 1 + z^{-1} + z^{-2} + z^{-3} + \cdots$$

$$= \frac{1}{1 - z^{-1}} = \frac{z}{z - 1}$$

Ex: $x(t) = e^{at}$

$$X(z) = \sum_{k=0}^{\infty} e^{akT} z^{-k}$$

$$= 1 + e^{aT} z^{-1} + e^{2aT} z^{-2} + \cdots$$

$$= \frac{1}{1 - e^{aT} z^{-1}} = \frac{z}{z - e^{aT}}$$

Ex: $x(t) = \delta(t)$

$$X(z) = \sum_{k=0}^{\infty} \delta(kT) z^{-k} = 1 + 0 + 0 + \cdots = 1$$

Ex: $x(t) = a^{t}$

$$X(z) = \sum_{k=0}^{\infty} a^{kT} z^{-k}$$

$$= 1 + a^{T} z^{-1} + a^{2T} z^{-2} + \cdots$$

$$= \frac{1}{1 - a^{T} z^{-1}} = \frac{z}{z - a^{T}}$$

Ex: $x(t) = t$

$$X(z) = \sum_{k=0}^{\infty} kT \, z^{-k}$$

$$= 0 + Tz^{-1} + 2Tz^{-2} + \cdots$$

$$z X(z) = T + 2Tz^{-1} + 3Tz^{-2} + \cdots$$

$$z X(z) - X(z) = T + Tz^{-1} + Tz^{-2} + \cdots$$

$$X(z)\,(z-1) = \frac{T}{1-z^{-1}} = \frac{Tz}{z-1}$$

$$X(z) = \frac{Tz}{(z-1)^2}$$

Ex: $x(t) = \sin(\omega t) = \dfrac{e^{j\omega t} - e^{-j\omega t}}{2j}$

$$X(z) = \sum_{k=0}^{\infty} \sin(\omega kT)\, z^{-k}$$

$$= \sum_{k=0}^{\infty} \frac{1}{2j}\left[e^{j\omega kT} z^{-k} - e^{-j\omega kT} z^{-k}\right]$$

$$= \frac{1}{2j}\left[\left(1 + e^{j\omega T} z^{-1} + e^{2j\omega T} z^{-2} + \cdots\right) - \left(1 + e^{-j\omega T} z^{-1} + e^{-2j\omega T} z^{-2} + \cdots\right)\right]$$

$$= \frac{1}{2j}\left[\frac{1}{1 - e^{j\omega T} z^{-1}} - \frac{1}{1 - e^{-j\omega T} z^{-1}}\right]$$

$$= \frac{1}{2j} \; \frac{\cancel{1} - e^{-j\omega T} z^{-1} \cancel{-1} + e^{j\omega T} z^{-1}}{1 - e^{j\omega T} z^{-1} - e^{-j\omega T} z^{-1} + z^{-2}}$$

$$= \frac{1}{2j} \; \frac{z\left(e^{j\omega T} - e^{-j\omega T}\right)}{z^2 - z\left(e^{j\omega T} + e^{-j\omega T}\right) + 1}$$

$$X(z) = \frac{z\left(\frac{e^{jwT} - e^{-jwT}}{2j}\right)}{z^2 - 2z\left(\frac{e^{jwT} + e^{-jwT}}{2}\right) + 1}$$

$$= \frac{z \sin(wT)}{z^2 - 2z\cos(wT) + 1}$$

Ex: $x(t) = \cos wt$

$$X(z) = \frac{z(z - \cos(wT))}{z^2 - 2z\cos(wT) + 1}$$

Properties of Z.T. :

1. $f_1(t) \pm f_2(t) \xrightarrow{Z.T} f_1(z) \pm f_2(z)$

2. $a f(t) \xrightarrow{Z.T} a f(z)$

3. $e^{\pm at} f(t) \xrightarrow{Z.T} f(z e^{\mp aT})$!! عكس الإشارة !!

4. $a^t f(t) \xrightarrow{Z.T} f\left(\frac{z}{a^T}\right)$

5. $t f(t) \xrightarrow{Z.T} -Tz \frac{df(z)}{dz}$

6. initial value

$$f(0) = \lim_{t \to 0} f(t) = \lim_{z \to \infty} f(z)$$

7. Final value

$$f(\infty) = \lim_{t \to \infty} f(t) = \lim_{z \to 1} (z-1) f(z)$$

$\underline{Ex}$: $y(k+2) + 3y(k+1) + 2y(k) = \delta(k)$

if $y(0) = 0$, $y(1) = -1$, Solve For $y(k)$

→ using Z.T.

$$z^2 y(z) - z^2 \cancel{y}(0) - z y(1) + 3[z Y(z) - z \cancel{y}(0)]$$

$$+ 2 Y(z) = 1$$

$$z^2 Y(z) + z + 3z Y(z) + 2 Y(z) = 1$$

$$Y(z) [z^2 + 3z + 2] = 1 - z$$

$$Y(z) = \frac{1-z}{z^2+3z+2} = \frac{1-z}{(z+1)(z+2)}$$

$$= \frac{A}{z+1} + \frac{B}{z+2}$$

$A = 2$

$B = -3$

$$Y(z) = \frac{2}{z+1} - \frac{3}{z+2} = 2z^{-1} \frac{z}{z+1} - 3z^{-1} \frac{z}{z+2}$$

→ Inverse Z.T.

$$y(k) = 2 (-1)^{k-1} u(k-1) - 3 (-2)^{k-1} u(k-1)$$

Ex: $$F(z) = \frac{z(z+1)}{(z+2)(z+4)}$$

$$= z\left[\frac{A}{z+2} + \frac{B}{z+4}\right]$$

$$A = \frac{-1}{2} \qquad B = +\frac{3}{2} = 1.5$$

$$= \frac{-1}{2}\frac{z}{z+2} + 1.5\frac{z}{z+4}$$

$$F(k) = \frac{-1}{2}(-2)^k u(k) + 1.5(-4)^k u(k)$$

Report. $$F(z) = \frac{z(z+1)}{(z+2)(z+4)}$$ for $T = 0.5$ sec